

## Answers to Homework 2

Due Friday, March 14.

Show all work.

You may work with others if you like, but please list their names if you do.

Toby

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Name

1) Suppose you put a piece of salt into one end of a sealed, saturated, horizontal soil column. Describe the resulting process(es) and the ultimate end product.

*The salt will dissolve, and locally decrease the total water potential (via the osmotic potential). This will be slightly compensated by a movement of water to this end of the column. But because the column is sealed and water is relatively incompressible, the actual mass movement will be negligible.*

*The main process will be diffusion, and the ultimate end product will be a column that is uniformly saturated with salty water.*

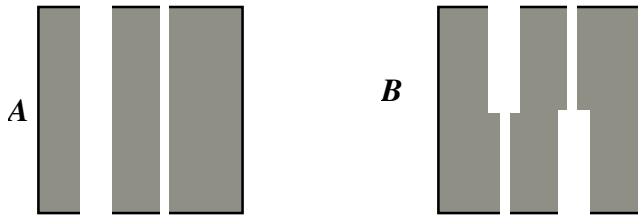
2) Do the same as #1, but for an **unsaturated** sealed horizontal soil column (say,  $\Theta = S$  (Hillel notation) = 0.5). Is there a semi-permeable membrane in the soil? If so, where? If not, is osmotic potential important?

*Similar to #1, but now water can move somewhat. There will be some mass flow toward the salt end, due to a drop in osmotic potential as the salt dissolves. This flow will eventually dissipate as the salt diffuses. Bottom line: diffusion is still the most important process, and the end result (a uniformly salty column) is the same (except for the degree of saturation).*

*The osmotic potential is a short-term player in moving water unless there is a semi-permeable membrane to prevent the salt from moving. Air is a good membrane: water vapor travels through air, but salt does not. But if there is continuity in the liquid phase (for example, water films are connected through the whole soil), the salt will eventually diffuse to equilibrium. Clay layers can also function as semi-permeable membranes as long as they don't have cracks, and they block the solute (either by charge, or by size). Clay liners will eventually leak, so the very-long-term result will be the same, but they will greatly slow the process of diffusive equilibration.*

3) You have 2 blocks of an impermeable metal, each one measuring  $1 \text{ cm}^3$  with 2 holes drilled through it. In block A, one hole has radius 1 mm, and the other has radius 0.1 mm. In block B, one hole has radius 1 mm halfway through, then radius 0.1 mm the rest of the way; the other hole is the opposite (first small, then large). What are the hydraulic conductivities of the two blocks (hint: note the similarity to a question on Exam 1)? How does this relate to soil structure? How does it not relate to soil structure (that is, what difference(s) between soil and solid metal are relevant to this question)?

*The left-hand side of Darcy's law can be written as  $q$ , a flux density, or  $Q/A$ , a total flux per unit area (that's the definition of flux density, and the units  $[L^3 T^{-1} / L^2]$  reduce to a velocity  $[L T^{-1}]$ ). We only have 2 holes per block, and we know  $A = 1 \text{ cm}^2$ , so let's work out  $Q$ , the total flux.*



Poiseuille's law says  $Q = \frac{\pi R^4}{8\eta} \frac{d\psi}{dx}$ , and we'll take the hydraulic gradient as unity (1.0). For Block A, it is easy to add the two holes together to solve for K for the whole block:

$$K = \frac{\pi[R_1^4 + R_2^4]}{8A\eta} = \frac{\pi[(0.001m)^4 + (0.0001m)^4]}{8(0.01m)^2(1.002 \times 10^{-2} \text{ kg} \cdot \text{m/s})} = 3.9 \times 10^{-7} \text{ m/s}$$

Block B's two holes have the same total flux, so we only need to solve for one of them, then multiply the result by two. You can think of one hole as being like a layered soil, and Hillel says that flow through a layered soil is just like flow through a uniform soil, except that there is conservation of mass:  $Q$  through the first layer =  $Q$  through the second layer. But for constant flux, if  $K$  decreases then the gradient must increase. In fact,  $K_{big} = 10,000 K_{small}$ , because  $K \sim R^4$  and so a factor of 10 difference in  $R$  means a factor of 10,000 difference in  $K$ . So we can essentially ignore the potential drop across the big portion of a Block B tube.

This means that flow through Block B is controlled by 2 small tubes, each one with twice the hydraulic gradient of Block A. So

$$K = 4 \frac{\pi R^4}{8A\mu} = \frac{\pi(0.0001m)^4}{2(0.01m)^2(1.002 \times 10^{-2} \text{ kg} \cdot \text{m/s})} = 1.56 \times 10^{-10} \text{ m/s}$$

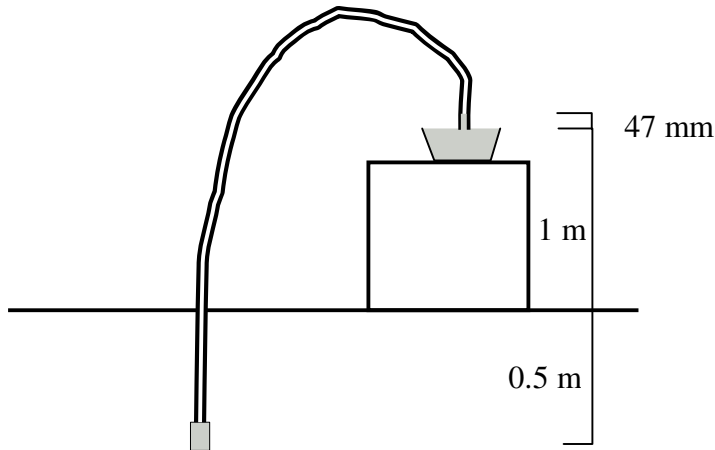
The ratio  $K_{block A}/K_{Block B}$  is essentially 10,001/4, or 2500 to 1. (Do you see how you could obtain this ratio without going through the actual numbers?)

Relationship with soil structure: what controls the overall conductivity is not just the presence of large pores, but whether they are continuous. The two blocks have identical porosity and pore size distribution, but their conductivities differ by about 2500. Unrealistic aspects: this "soil" only has two pores, and we know everything about them that we need to. Real soil is way more complex, the pores have strange shapes, they are multiply interconnected, and they can't readily be counted.

4) A given soil has  $K_{sat} = 3 \text{ cm/hr}$ , and  $\theta_s = \text{porosity} = 0.5$ . Plot  $K(\theta)$  from  $\theta = 0.0$  to 0.5 for the case of A:  $K(\theta) = K_{sat} * (\theta / \theta_s)$ , and B:  $K(\theta) = K_{sat} * (\theta / \theta_s)^4$ . You conduct an unsaturated hydraulic conductivity test by applying water slowly at the top, and letting it drain freely at the bottom (in other words, you are not using air pressure, or ponded water, to push the water out the bottom of the column). Why is the total potential gradient NOT linear?

(Skipping the graphs here...) In steady-state, vertical unsaturated flow, the water content varies from top to bottom. But the flux is constant, so the gradient must be greater in less saturated regions and less in wetter regions. In other words, the gradient is not linear.

5) A tensiometer is buried at a depth of 50 cm in a soil with a water table at 3 m. A thin tube comes up out of the tensiometer, and goes into a container of mercury (density 13.6 g/cm) whose upper surface is 1 m above the soil surface (see diagram). The system is at equilibrium when the mercury in the tube is 47 mm above the mercury surface. What is the matric potential of the soil around of tensiometer? Is water moving upward or downward in the soil between the tensiometer and the water table?



For a tensiometer, we assume the system is (effectively) at equilibrium. So the “pull” on one end of the liquid in the tube must equal the pull on the other end. Anything above the mercury level is the same on both sides, so it can be ignored. Quantitatively, using units of meters of water,

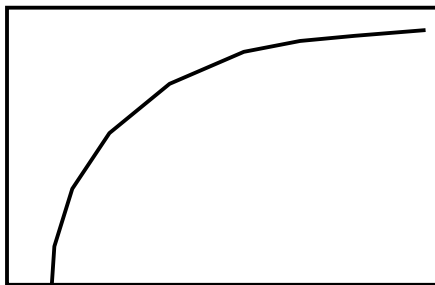
$$-\psi_m + h_{\text{leftside}} = \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} h_{\text{mercury}}$$

So we can rearrange, plug in numbers, and solve for matric potential:

$$\psi_m = -(13.6 \times 0.047) + 1.547 = 0.91m$$

Notice that this is positive (so it's pressure rather than tension): there is 91 cm worth of water pressure above a tensiometer buried 50 cm deep! Either your field is flooded, or you have a bad reading. If the reading is good, then water is flowing downward from the level of the tension.

6) Water with a non-reactive neutral tracer flows into a soil saturated with tracer-free water. You collect water at a depth of 2 m and see a breakthrough curve like this:

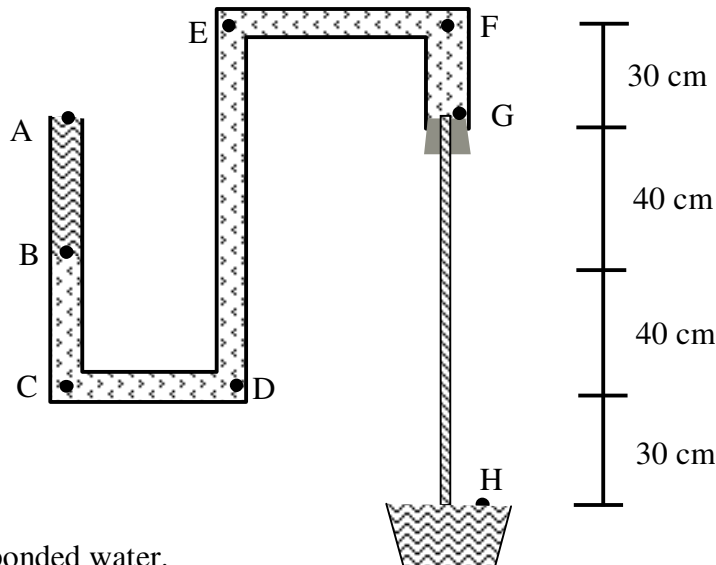


What conceptual model does this correspond to? What can you conjecture about the retardation? Should you expect the standard deviation of the tracer spread to increase with depth, or with the square root of depth?

*This is the curve you would see in fast flow through a single tube, before diffusion had set in to distribute solute better between the fast streamlines in the tube center and the slow streamlines at the wall. You won't see curves like this from most soils, and you should also not expect a curve to tell you, unambiguously, what model best fits it!*

*The ADE will give an S-shaped curve, with a "foot" at the left bottom. This one rises abruptly, so you can eliminate the ADE. But I believe that both the streamtube and the MIM model can give a curve like this. For the streamtube model, the standard deviation would scale linearly with time and there would be retardation; for the MIM it would scale with the square root of time and retardation could be significant. Assuming a constant velocity downward, depth-scaling would be the same as time-scaling.*

7) You have the following setup:



A to B is ponded water.

B to G is saturated soil.

G to H is a water-filled tube.

Distance from C to D: 40 cm.

Distance from E to F: 60 cm.

Use point B as your gravitational reference level.

Assuming the system is steady-state, make a table of  $\psi_T$ ,  $\psi_g$ , and  $\psi_m$  for points A through H. If air leaked into the system at the rubber stopper at G, what additional information would you need (and how would you obtain it) to determine  $K(\theta)$  between E and F?

Following the method I gave earlier, make a table and fill it in in this order:

- You know everything you need to fill in the gravitational potential column.
- You have what you need to get A, B, G, and H for the matric potential column.

- Darcy's law tells you that, for saturated flow through a uniform soil, the total potential gradient will be linear between B and G.
- Now you can calculate matric potentials given the total and gravitation potentials.

Note that the convention is to suppose that the water-filled parts of the tube are at equilibrium. That is, you should assume that the total potential is the same at A as at B, and the same at G as at H. Why? Think back to problem 3, with the different holes through the block. A water-filled tube has vastly greater conductivity than a soil-filled tube. Remember that  $q_{\text{water}} = q_{\text{soil}}$  so  $K_{\text{water}} * \text{Gradient}_{\text{water}} = K_{\text{soil}} * \text{Gradient}_{\text{soil}}$ . So you can assume that all the potential drop will be "used up" in the soil, and none in the water-filled portion of the tube.

So, the Table. Units are in cm of water.

Location:	Total Potential =	Gravitation Potential +	Matric Potential
A	40	40	0
B	40	0	40
C	24.3	-40	64.3
D	8.6	-40	48.6
E	-34.6	70	-104.6
F	-58.2	70	-128.2
G	-70	40	-110
H	-70	-70	0

Inside the tube, the total potential gradient is  $110/280 = 0.393 \text{ cm/cm}$ .

If air leaked into the system at G, you'd have unsaturated flow at E, F, and G. In a worst-case situation, enough air would leak in to break the hanging water column at G..H, and flow would actually reverse. But let's ignore that, and see what we'd need to calculate  $K(\theta)$  between points E and F:

- Flux between E and F. If we wait long enough, the system will come to equilibrium, and we can get this at A or H.
- Cross-sectional area of the tube, so you can convert the flux to a flux density.
- The actual water content of the E..F section. Because water content is unlikely to be constant in this range (why? You have a potential gradient, and that will cause a water content gradient), assuming a single value for this interval may not be a good assumption. As a rule, the larger the interval over which you measure a non-linear parameter, the poorer the averaging.
- The actual potentials at E and F. You'll have to find a way to measure these without further disturbing the system. A tensiometer would be a good choice.