Today:
Water in the unsaturated zone

Lecture 10

Recharge

The atmosphere is unsaturated. If you dig a hole, you need to get to a certain depth before it starts filling with water.

So before it can be part of an aquifer, water must move through unsaturated material.

Infiltration: the movement of water from above ground surface to below ground surface

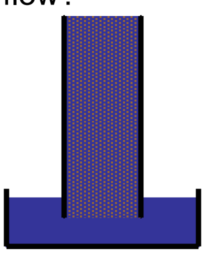
Recharge occurs when water reaches an aquifer

Let's examine the energy of water in unsaturated flow

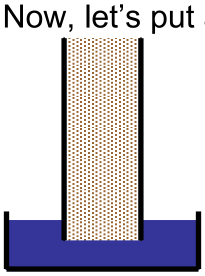
Gravitational potential energy (same as for saturated) = gz

Is elastic (pressure) potential the same?

Imagine we placed a saturated sand column upright in a pan of water. Would there be any flow?

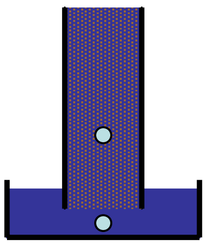


Now, let's put a column filled with *dry* sand into the pan of water. Would there be any flow?

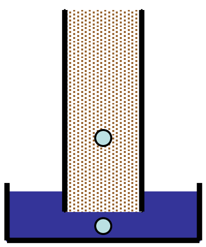


Why would flow move in these directions?

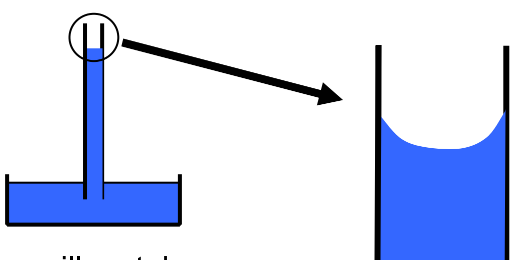
SATURATED COLUMN: How does total head at the two points shown compare (use the pan bottom as the datum)?



UNSATURATED COLUMN: How does total head at the two points shown compare (use the pan bottom as the datum)?



So, for saturated flow, P ≥ 0; for unsaturated flow, P < 0. The negative pressure is the result of capillary forces



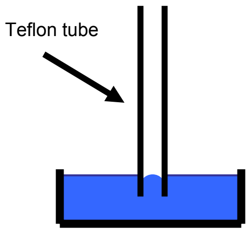
capillary tube in water

The meniscus results from:

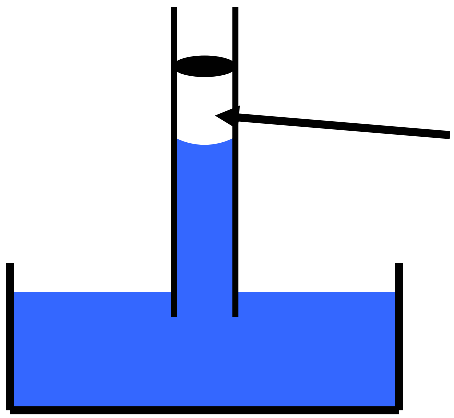
- Adhesion
- Cohesion
- Surface tension

Water weight that is in the center of the column is not supported by adhesion—it is supported by surface tension.

Surface tension results because water is polar. At the air-water interface, the unsatisfied bonds form hydrogen bonds that form a net.

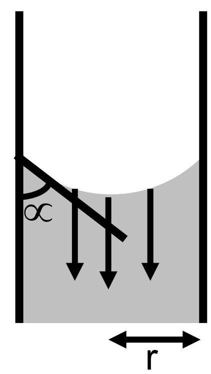


If we used a Teflon tube instead of a glass tube, we would have an 'inverse' meniscus because water can't adhere well to Teflon



In a regular tube, place a disk above the meniscus and measure pressure between disk and meniscus.

It will be negative.

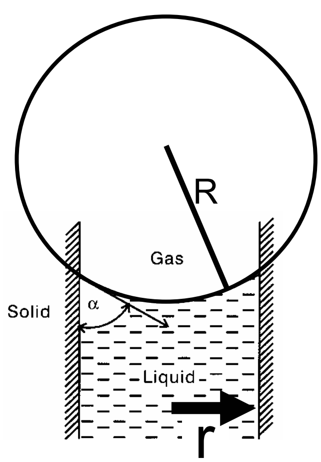


Why the meniscus? Water molecules adhere to walls of capillary tube, and the water molecules at the air-water interface for a net that supports the weight of the water column hanging below (surface tension).

Why is pressure negative? Cohesive forces cause water to hold on to the 'net' at the interface formed by surface tension. The weight of this water pulling down on the net is what causes the negative pressure.

We want to find P_{cap}

$P_{cap} = P_{atm} - P_{H_2O}$, but $P_{atm} = 0$, so $P_{cap} = - P_{H_2O}$



The radius of a circle that fits the curve of the meniscus is called R, the radius of curvature.

$$\frac{\cos(\propto)}{r} = \frac{1}{R}$$

σ: surface tension between the liquid and the air

$$P_{cap} = \frac{2\sigma\cos(\propto)}{r} = \frac{2\sigma}{R}$$

As r or R decreases (or $\propto$ decreases), P_{cap} increases

Water will rise in the tube in response to pressure decrease caused by the meniscus until the weight of the water below counters the negative pressure.

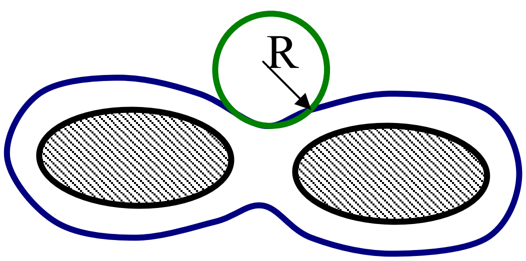
$$pgh = P_{cap}$$

$\rho g = \gamma$ (specific weight)

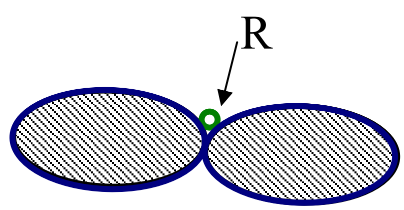
$$\gamma\, h = P_{cap} = \frac{2\sigma}{R} = \frac{2\sigma\cos(\propto)}{r}$$

$$h = \frac{2\sigma}{\gamma\, R} = \frac{2\sigma\cos(\propto)}{\gamma\, r}$$

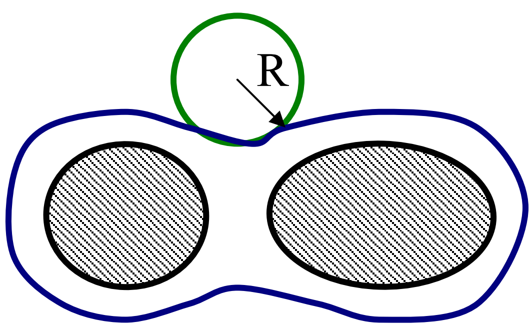
So as r (or R) decreases, the magnitude of h increases



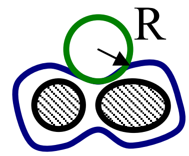
Wet soil



Dry soil



Coarse soil



Fine soil

R is much smaller for dry soils than wet soils, and for fine soils than coarse soils, so P_{H_2O} will be more negative for dry soils and fine soils than wet soils and coarse soils.

What makes water flow?
So far, we have covered

- -Gravitational potential (same as saturated)
- -Pressure potential (pressures in unsaturated porous media are negative, not positive, and depend on wetness and soil type)

Other Potentials:

- -Chemical potential (osmotic forces)
- -Temperature potential
- -Electrical potential

Just as in saturated flow we typically ignore these, as they are usually insignificant.

So total potential is:

$$\Phi_{unsat} = gz + \frac{\psi}{\ell} \qquad \Phi_{sat} = gz + \frac{p - p_0}{\ell}$$

ψ is "matrix suction" or "matric tension" (negative pressure) $\left[\frac{N}{m^2}\right]$

$$h = z + \frac{\psi}{\gamma} = z + \Psi \qquad \text{where} \quad \Psi = \frac{\psi}{\gamma}$$

Ψ is the suction head [L], "tension head" (just pressure head with a negative value)

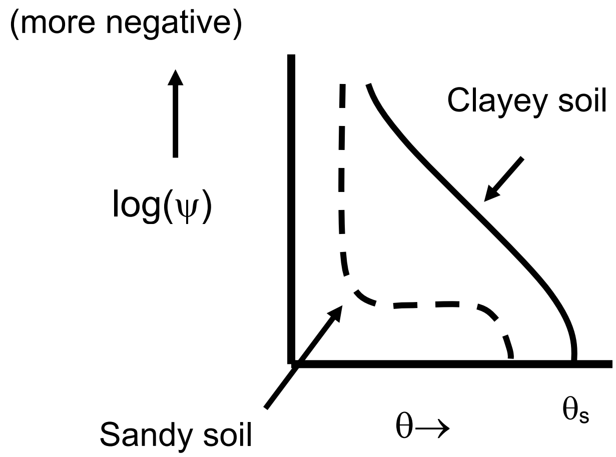
ψ (little psi) depends on:

- grain size and shape
- degree of wetness

Degree of wetness is described by θ, the volumetric water content

$$\theta = \frac{\text{volume of water}}{\text{total volume of soil}}$$

If saturated (θ_s), θ = n



One "fit" equation

$$\psi = \frac{a(n-\theta)^b}{\theta^c}$$

a, b, c are empirical constants

Storage in unsaturated materials increases as water content increases—Water in storage = θ.

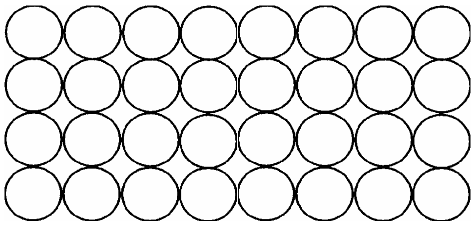
A plot of (ψ vs. θ) is called "water retention curve"—essentially a plot showing what portion of the pore volume has a given matrix suction.

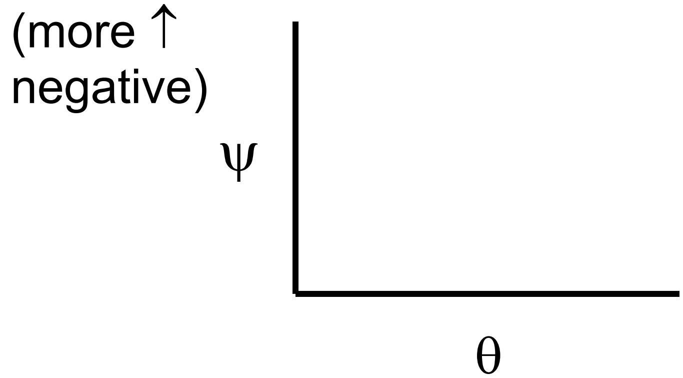
If water is in a pore in the vadose zone, under what condition will it flow out of the pore?

Let's assume we're looking at flow from a pore to a point of equal elevation is outside the pore. Now water will move out of a pore only if

Suppose instead of soil, we have a bundle of capillary tubes, all with the same diameter (and all filled with water held in by capillary forces). We will try to get the water out of the tubes with a miniature vacuum cleaner. Assume that this vacuum cleaner has a dial that controls how hard it sucks.

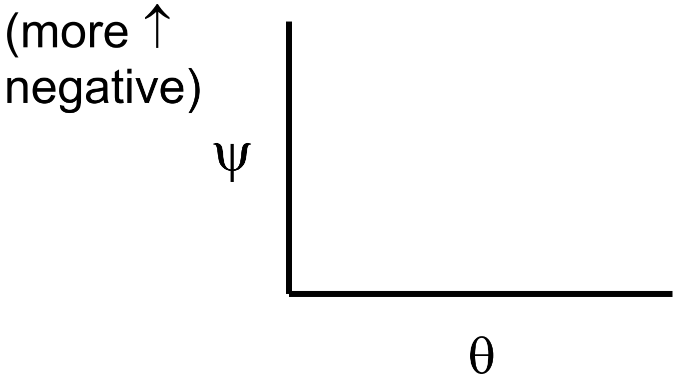
Put the nozzle up to the first capillary tube, and start at a very low suction. Assuming the suction is low enough, no water will leave the tube. Now, slowly increase the suction until water flows from the first tube. At this same pressure will it flow from the rest?

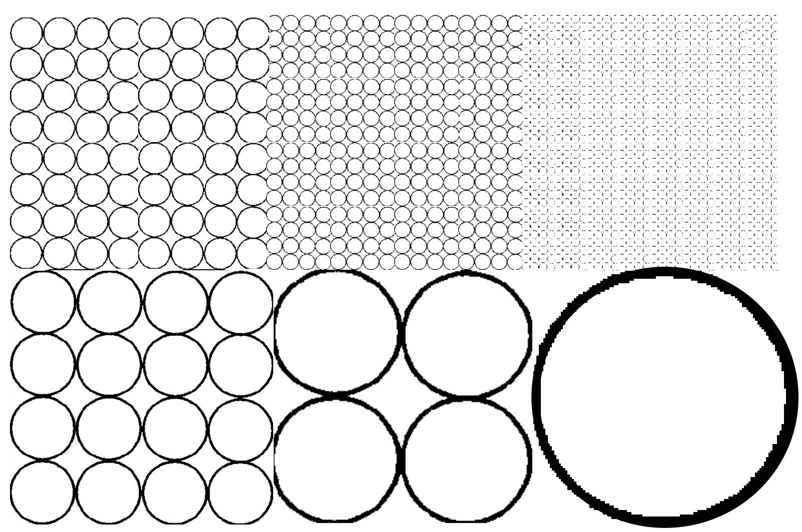




Now, imagine our collection of capillary tubes is different: instead of each tube having the same r, there are ‘bundles’ of tubes. Each bundle is composed of capillary tubes with the same r, but each bundle’s r value is unique. In addition, the bundles are scaled so that the volume of water in each bundle is equal. The vacuum cleaner nozzle is now as big as a bundle of tubes.

Using the same procedure as before, we put the nozzle up to the bundle with the biggest r value, starting at a low suction. We then increase suction until the water flows out of all the tubes in the bundle (that happens at the same time, since they all have the same r). At the same suction, will the bundle with the next-largest r drain?

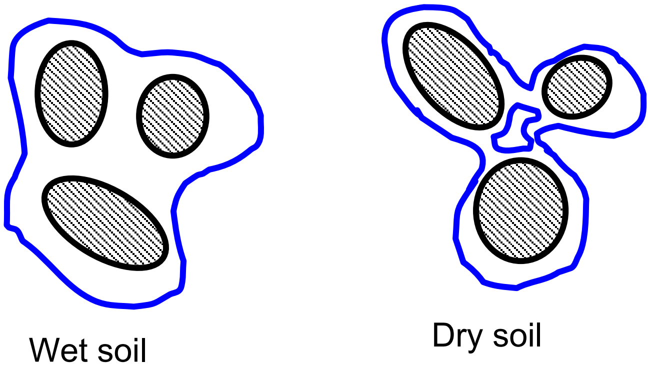




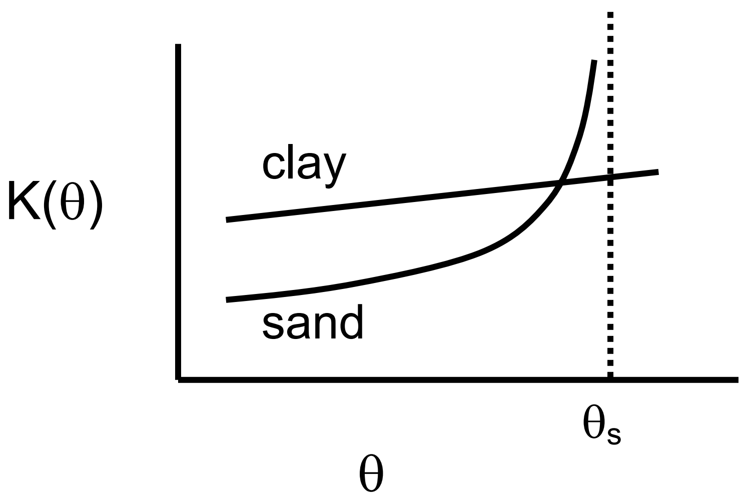
Back to our original question—why do these water retention curves look different?

The sandy soil curve looks similar to the scenario where all the capillary tubes had the same r; the clayey soil curve resembles the one where each bundle of tubes had a different r. In fact, the pores in sands that are reasonably well sorted typically cluster around one size; clays typically have a much more even distribution of pore sizes.

<u>Hydraulic Conductivity of Unsaturated Materials</u>



There is less cross-sectional area available for flow in dry soils, so K decreases as θ decreases.



At low θ, a larger proportion of clay pore center is filled.

One "fit" equation: $K = \dfrac{a}{b + \psi^m}$

a,m empirical

At low θ, K is higher for clay than sand!

Equations for unsaturated flow:

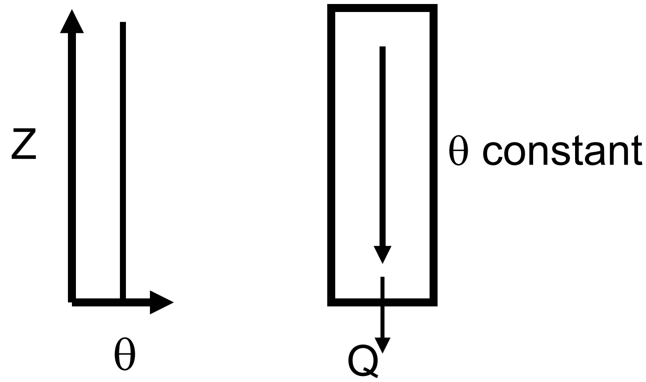
$$\text{Continuity}: \quad -\nabla \bullet q = \frac{\partial \theta}{\partial t}, \ \text{non-linear in } \theta$$

$$\text{General flow}: \quad \nabla \bullet \left[K(\theta) \nabla \left(z + \frac{\psi \theta}{\gamma} \right) \right] = \frac{\partial \theta}{\partial t}$$

To model flow in the unsaturated zone, we must know the distribution of θ, and the relationships between K and θ, ψ and θ.

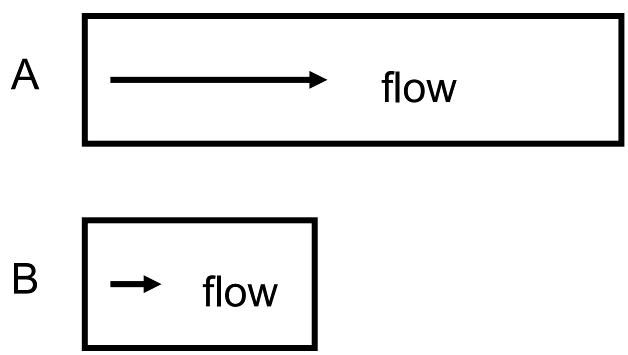
__**Various points of interest for unsaturated flow**__

1.) For uniform θ in uniform media,
$\nabla\psi = 0$; $\nabla h = \nabla z = 1$ (unit gradient); $\dfrac{\partial\theta}{\partial t} = 0$

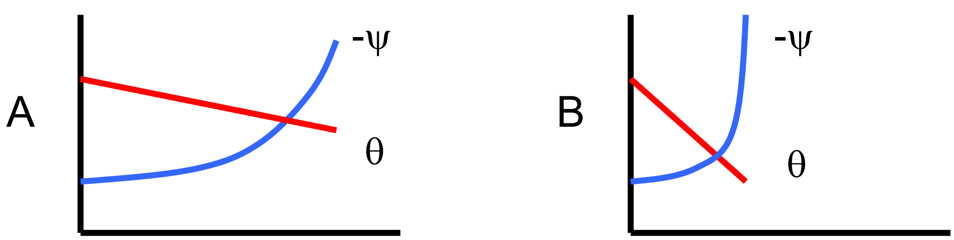


Only gravitational potential is significant.

2.) Varying θ in homogeneous media

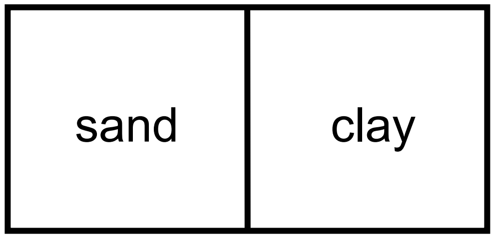


A + B have the same total variation in θ, but $\nabla\psi$ is much larger for B than A.

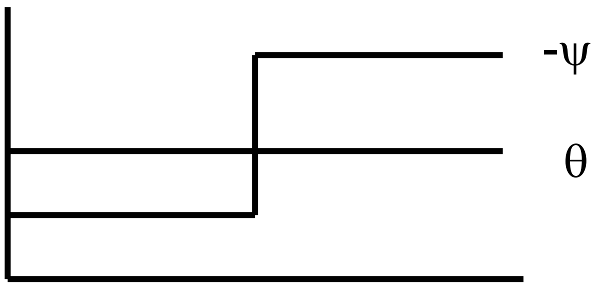


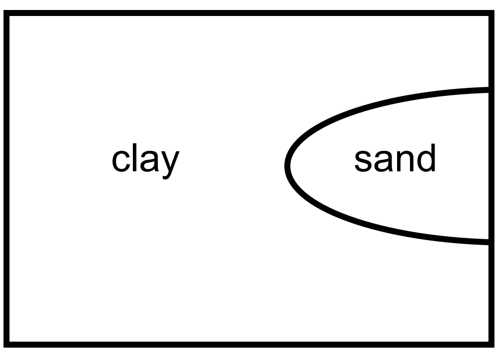
Soil moisture "fronts" cause greater fluxes than gradual moisture changes.

3.) For heterogeneous media

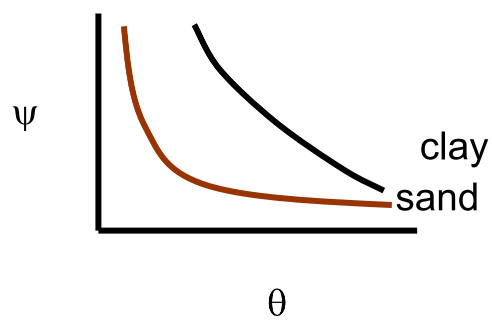


ψ is more negative for clay, so water will move from sand to clay





How will water flow (L $\rightarrow$ R)?



ψ is less negative in sand. Gradient is toward clay.